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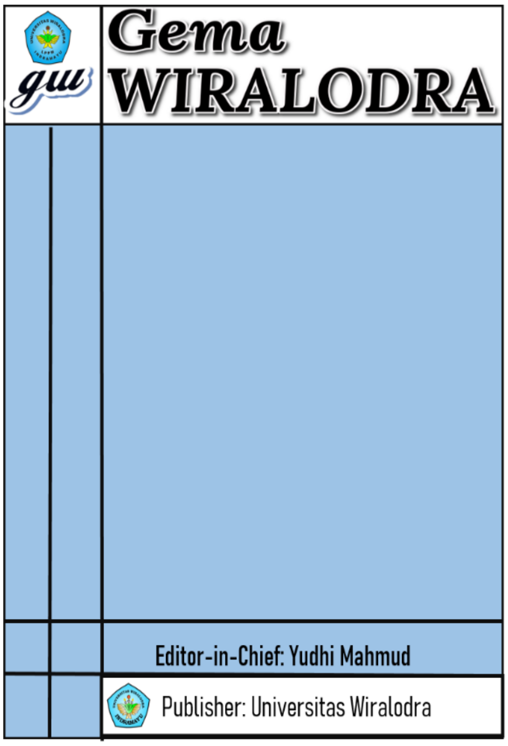

Application of *the geometric Brownian motion model in West Texas Intermediate* crude oil price prediction

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Orginal Article

Application of *the geometric Brownian motion model* in *West Texas Intermediate* crude oil price prediction

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Abstract
Crude oil is one of the primary commodities in the global economy. Crude oil prices are among the most complex and challenging to model because of the irregular, non-linear, non-stationary fluctuations in crude oil prices and their high volatility. It is essential to predict crude oil commodity prices to reduce the negative impact of changes in crude oil commodity prices. Several mathematical models can be used to forecast crude oil commodity prices. One model that can be used is the *Geometric Brownian Motion model*, also known as the *Wiener process*. In this research, predictions for WTI (*West Texas Intermediate)* crude oil in 2022 were carried out using the *Geometric Brownian Motion model*. The results of this research are predictions of crude oil prices for July 2023 with iterations of 100, 200, and 1000, respectively, producing MAPE values of 6.092415%, 7.364198%, and 7.276606%.
Keywords: Crude Oil, Stochastic Process, *Geometric Brownian Motion*

1. Introduction

Global economic development is always closely related to the existence of energy. The availability of energy, as an essential component in the production process, supports economic growth. One of the primary commodities for the global economy today is crude oil. World crude oil commodity prices are divided into two types, namely the *Brent type* and *the West Texas Intermediate* (WTI) type. The *Brent* type is the standard value of crude oil originating from the North Sea of Europe, and the WTI type is the standard value of crude oil produced in Texas, United States (Carollo, 2012)

Crude oil prices are among the most complex and challenging to model because of the irregular, non-linear, non-stationary fluctuations in crude oil prices and their high volatility. Therefore, it is essential to predict crude oil commodity prices to reduce the negative impact of fluctuations in crude oil commodity prices. Applied mathematical models, artificial intelligence, *big data*, and forecasting models are used to predict future crude oil commodity prices (Hendrawaty et al., 2020). One model that can be used is the *Geometric Brownian Motion model*, also known as the *Wiener process*. The *Geometric Brownian Motion* Model is a continuous time stochastic model with random variables following *Brownian motion* (Reddy & Clinton, 2016).

Previous research used the GBM model to predict asset prices such as options, shares, and other commodities. Bayun et al. (2022) applying the GBM model to forecasting sharia stock prices produces a good level of accuracy, indicated by a MAPE value of 1.87%. Izzata (2018), in his research, applied the GBM model with Ito's Lemma to predict stock prices, resulting in a good level of accuracy, as indicated by a MAPE value of less than 5%. In his research, Bogdana used geometric Brownian motion to forecast crude oil prices with *a particular emphasis* on macroeconomic factors, resulting in a MAPE value of 23.677% (Pereboichuk, 2013).

Crude oil

Crude oil is a naturally occurring and flammable oily liquid, consisting of various organic and inorganic chemicals that are found in large quantities beneath the earth's surface (Pereboichuk,

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2013). Crude oil price movements are a stochastic process because the dynamics of price movements are uncertain. Crude oil price fluctuations are influenced by *drit* and volatility parameters. The formula for crude oil (R_t)*returns* is defined as follows:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

With $P_t$is the price of crude oil at time $t$and $P_{t-1}$is the price of crude oil at time $t-1$.

***Geometric Brownian Motion* (GBM)**

Brownian motion or commonly called the Wiener process is a continuous stochastic process. Brownian motion is formed from a symmetric random walk equation by finding the limit value of the random walk.

Definition:

Ross (1966) defines a stochastic process $\{W(t): t \geq 0\}$ where the function $W: \mathbb{R}^+ \to \mathbb{R}$is said to be a Brownian motion process if

1) $W(0) = 0$
2) $\{W(t), t \geq 0\}$has *stationary* and *independent increments*
3) For each $t > 0$, $W(t)$it is normally distributed with mean 0 and variance $\sigma^2 t$.

Brownian motion is divided into 2 types: Brownian motion with drift and Geometric Brownian motion. If $\{X(t), t \geq 0\}$it is a Brownian motion process with coefficients $\mu$and parameters σ^2, then the process $\{W(t), t \geq 0\}$is defined as $W(t) = e^{X(t)}$geometric Brownian motion.

In general, the GBM model is stated as follows:

$$dX = \mu X dt + \sigma X dW$$

With Xcrude oil prices, μ*drift* values, σ volatility values, and $dW$are changes in the Wiener process. For example, $\ln X = F(X,t)$, $\mu X = \mu(X,t)$and $\sigma X = \sigma(X,t)$. Crude oil price fluctuations can be defined using the stochastic differential equation as follows:

$$dX = \mu(X,t)dt + \sigma(X,t)dW$$

It is assumed that X following Ito's Lemma. so the equation becomes as follows (Dmouj, 2006)

$$d(\ln X) = \left(\frac{\partial(\ln X)}{\partial X}\mu(X,t) + \frac{\partial(\ln X)}{\partial t} + \frac{1}{2}\frac{\partial^2(\ln X)}{\partial X^2}\sigma(X)^2\right)dt + \left(\sigma\frac{\partial(\ln X)}{\partial X}\right)dW_t$$

$$= \left(\frac{1}{X}\mu X + 0 + \frac{1}{2}\left(\frac{-1}{X^2}\right)\sigma(X)^2\right)dt + \frac{\sigma X}{X}dW_t$$

$$= \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dW_t$$

$$\ln X_t - \ln X_{t-1} = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dW_t$$

$$\ln X_t = \ln X_{t-1} + \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dW_t$$

$$e^{\ln X_t} = e^{\ln X_{t-1} + \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dW_t}$$

$$X_t = X_{t-1}e^{\left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dW_t}$$

With $X_t$is the current crude oil price prediction $t$and $X_{t-1}$is the current crude oil price prediction $t-1$.

***Drift* $(\hat{\mu})$Parameter Estimation and Volatility$(\hat{\sigma})$**

Volatility is the level of movement in crude oil prices. The formula for the volatility value is as follows (Tsay, 2006)

$$\hat{\sigma} = \frac{s_r}{\Delta t}$$

Where,

$$\bar{R} = \frac{\sum_{t=1}^{n} R_t}{n}$$

$$s_r = \sqrt{\frac{\sum_{t=1}^{n}(R_t - \bar{R})}{n-1}}$$

With $\bar{R}$is the average *return of* crude oil, $s_r$standard deviation of crude oil andΔt is the time interval in calculating the *return value*.

Drift is the expected rate of movement in crude oil prices. The formula for *drift* is as follows (Tsay, 2006)

$$\hat{\mu} = \frac{\bar{R}}{\Delta t} + \frac{\hat{\sigma}^2}{2}$$

Data Normality Test

Data normality testing can use the *Kolmogorov-Smirnov test* with a hypothesis

H_0: Sample data is normally distributed

H_1: Sample data is not normally distributed

And test statistics

$$D_{hitung} = maks|F_t - F_s|$$

$D_{hitung}$is the general deviation, $F_t$the distribution function which is hypothesized to have a normal distribution and $F_s$the cumulative distribution function of the sample data.

The test criteria are if $D_{hitung} < D_{\alpha,n}$it $H_0$is $\alpha = 0{,}05$accepted, meaning that the sample data is normally distributed (Razali & Wah, 2011).

***Mean Absolute Percentage Error* (MAPE)**

Based on Abidin (2013), *Mean Absolute Percentage* Error (MAPE) is the average absolute percentage of forecasting errors. MAPE is an important factor in evaluating forecasting accuracy. MAPE will show how big the forecast error is compared to the actual value. If the MAPE value resulting from a forecasting method is smaller, the forecasting method is getting better. The calculation of the MAPE value is as follows:

$$MAPE = \frac{\sum_{p=1}^{n}\left|\frac{P_t - F_t}{P_t}\right|}{n} \times 100\%$$

where $P_t$is the actual value at time t. $F_t$is the forecast value at time t. $n$is the amount of data observed.

Table 1

MAPE Accuracy Rating Scale

MAPE value	Forecasting Accuracy
< 10%	Forecasting accuracy is very good
11% - 20%	Good forecasting accuracy
21% - 50%	Forecasting accuracy is still within reasonable limits
> 51%	Forecasting accuracy is inaccurate

2. Methods

The research utilizes data from http://www.investing.com. Specifically, it uses WTI (West Texas Intermediate) crude oil price data. The data covers the period from January 2022

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to August 2022. In total, there are 156 historical data points for this period. The collected data is divided into two segments: In-sample data: This portion of the data is used for model development and analysis. Out-of-sample data: This portion is reserved for model validation.

Within the in-sample data, a statistical test called the Kolmogorov-Smirnov test is applied. This test assesses whether the returns on crude oil prices in the in-sample data follow a normal distribution. Normality is a common assumption in many statistical models. Using the in-sample data, parameter values are estimated. Two specific parameters are mentioned: $\hat{\mu}$ (mu-hat) and $\hat{\sigma}$ (sigma-hat). These parameters are likely essential for a statistical model, possibly the Geometric Brownian Motion model. The research employs the Geometric Brownian Motion model (a mathematical model often used in finance) to predict future crude oil prices. This model takes into account the estimated parameters ($\hat{\mu}$ and $\hat{\sigma}$) and uses them to simulate how crude oil prices might change over time. The validity of the predictive model is assessed. This is done by calculating the MAPE (Mean Absolute Percentage Error) value. MAPE measures how accurate the model's predictions are by comparing them to actual observed crude oil prices.

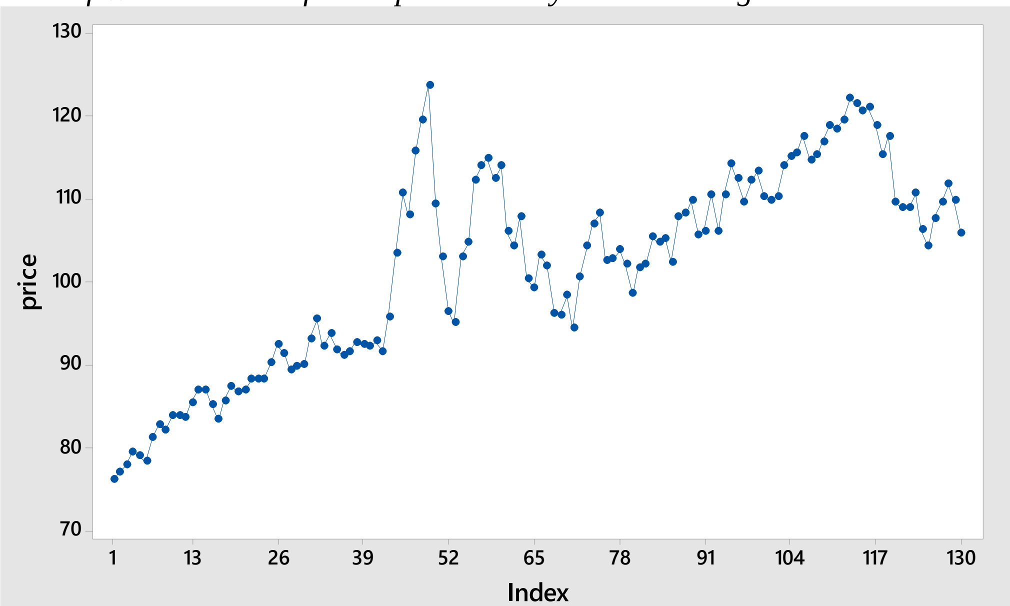
Based on the results of the model validation and other analyses conducted throughout the research, conclusions are drawn. These conclusions may pertain to the model's ability to predict crude oil prices during the specified time frame or other findings from the research. In summary, this research method involves collecting and analyzing WTI crude oil price data, testing its normality, estimating parameters, using a mathematical model to predict future prices, and assessing the model's accuracy. The final step is to draw conclusions based on these analyses, which may have implications for understanding and forecasting crude oil price movements.

3. Results and Discussion

The sample data consists of 130 crude oil price data with 129 crude oil price return data from 03 January 2022 to 30 June 2022. Meanwhile, the out-sample data consists of 26 crude oil prices from 1 July 2022 to 3 August 2022. The following is the crude oil price data used in the research.

Figure 1

Actual Price of WTI Crude Oil for the period 1 July 2022 to 3 August 2022



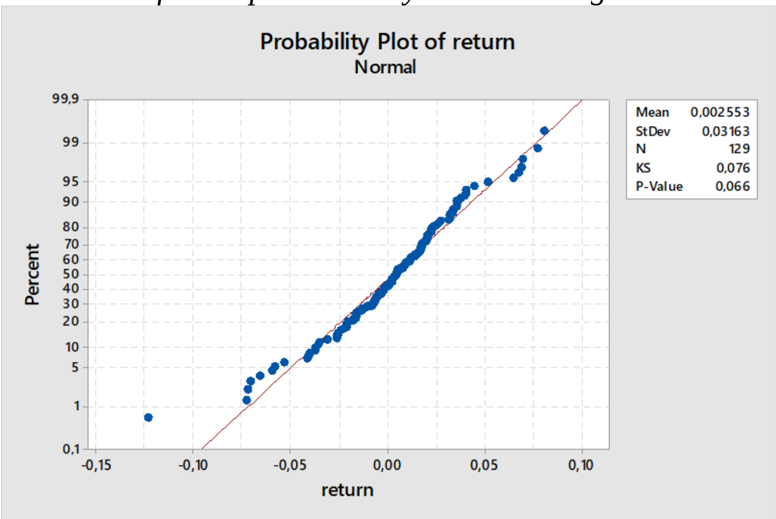
Normality test

The first stage carried out was to carry out a normality test on the crude oil *return value* using the *Kolmogorov-Smirnov test*. With a 95% confidence level $\alpha = 0{,}05$and test statistics

$$D_{hitung} = maks|F_t - F_s|$$
$$= 0{,}068479$$
$$D_{\alpha,n} = \frac{1{,}36}{\sqrt{129}}$$
$$= 0{,}119741$$

Kolmogorov-Smirnov test criteria, $D_{hitung} < D_{\alpha,n}$it can be concluded that crude oil returns are normally distributed. The following are the results of the crude oil return normality test using Minitab software.

Figure 2
WTI Crude Oil Price Return for the period 1 July 2022 to 3 August 2022



Parameter Estimation

Drift and volatility parameters are estimated from the *Geometric Brownian Motion model*. The first step is to calculate the average *return*$(\bar{R})$ and the standard deviation $(s_r)$of crude oil *returns* obtained values $\bar{R} = 0{,}002553$and $s_r = 0{,}03163$. Based on these values, it will then be used to calculate the *drift parameters*$(\hat{\mu})$ and volatility $(\hat{\sigma})$is obtained:

Table 2
GBM Model Parameter Estimation

Parameter	Mark
Drift$(\hat{\mu})$	0.066595
volatility$(\hat{\sigma})$	0.357887

Crude Oil Price Prediction

Using the Python program, crude oil price predictions were calculated with 100, 200 and 1000 iterations of the GBM model trajectory. The prediction results with each iteration are shown in table 3 below.

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Table 3
Crude Oil Price Prediction

Date	Actual Price	Prediction			Error		
		100 iterations	200 iterations	1000 iterations	100 iterations	200 iterations	1000 iterations
01/07/2022	108.43	108.96	108.43	106.32	0.004888	0.005072	0.01946
03/07/2022	108.19	107.97	108.57	104.28	0.002033	0.003512	0.03614
04/07/2022	110.4	105.43	110.4	106.68	0.045018	0	0.033696
05/07/2022	99.5	108.43	109.62	105.67	0.089749	0.101709	0.06201
06/07/2022	98.53	102.99	107.03	105.61	0.045265	0.086268	0.071856
07/07/2022	102.73	104.17	105.37	102.88	0.014017	0.025698	0.00146
08/07/2022	104.79	108.23	105.98	101.07	0.032828	0.011356	0.0355
07/11/2022	104.09	109.2	104.47	97.62	0.049092	0.003651	0.062158
07/12/2022	95.84	108.17	104.28	104.23	0.128652	0.088063	0.087542
07/13/2022	96.3	103.22	110.14	108.64	0.071859	0.143718	0.128141
07/14/2022	95.78	106.99	103.53	106.84	0.117039	0.080915	0.115473
07/15/2022	97.59	96.03	105.95	100	0.015985	0.085665	0.024695
07/18/2022	102.6	104.01	104.38	104.84	0.013743	0.017349	0.021832
07/19/2022	104.22	103.14	106.55	107.87	0.010363	0.022357	0.035022
07/20/2022	102.26	107.38	105.6	106.9	0.050068	0.032662	0.045375
07/21/2022	96.35	110.43	105.74	106.59	0.146134	0.097457	0.106279
07/22/2022	94.7	106.61	106.39	104.57	0.125766	0.123442	0.104224
07/25/2022	96.7	106.47	108.6	103.34	0.101034	0.123061	0.068666
07/26/2022	94.98	102.42	108.87	110.67	0.078332	0.146241	0.165193
07/27/2022	97.26	100.84	100.96	101.76	0.036809	0.035266	0.046268
07/28/2022	96.42	100.89	107.29	103.78	0.04636	0.112736	0.076333
07/29/2022	98.62	103.61	106.52	107.71	0.050598	0.080105	0.092172
07/31/2022	97.94	100.33	108.01	106.12	0.024403	0.102818	0.083521
01/08/2022	93.74	100.8	109.57	113.56	0.075315	0.168871	0.211436
02/08/2022	93.83	105.28	102.64	101.44	0.122029	0.093893	0.081104
03/08/2022	96.25	104.59	108.07	103.6	0.086649	0.122805	0.076364

Figure 3
Comparison Chart of Actual and Predicted WTI Crude Oil Prices (100 iterations)

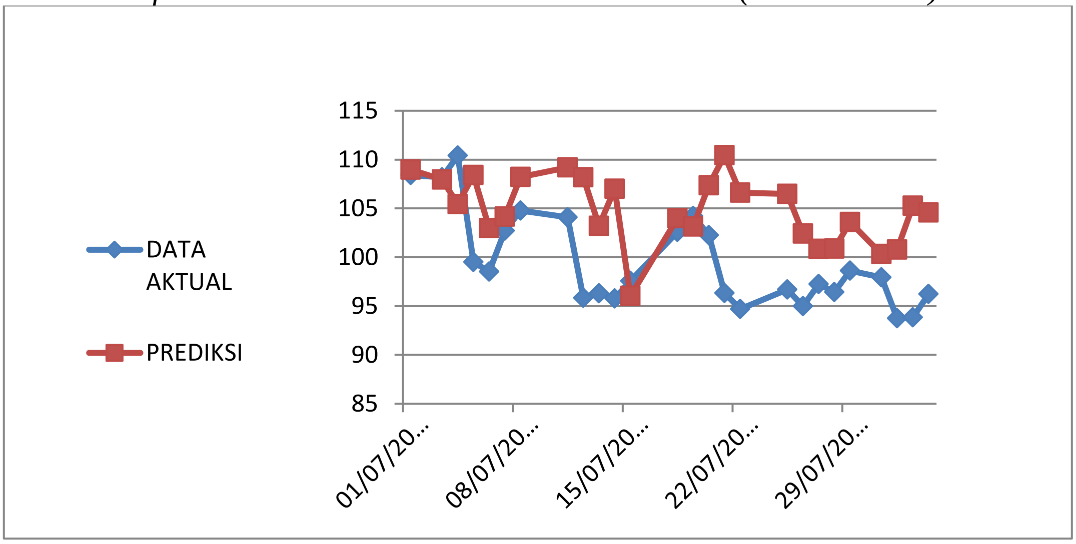

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Figure 4
Comparison Chart of Actual and Predicted Prices of WTI Crude Oil (200 iterations)

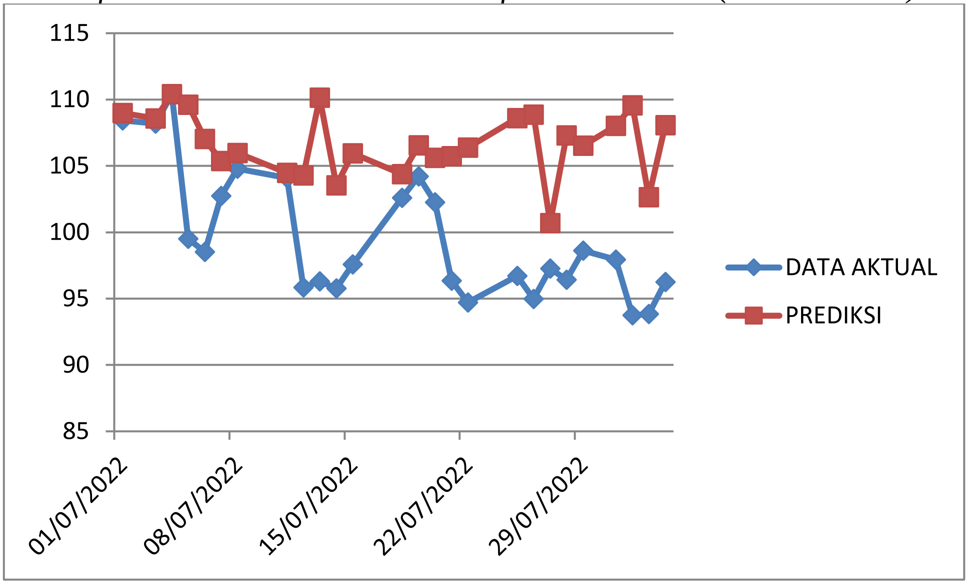
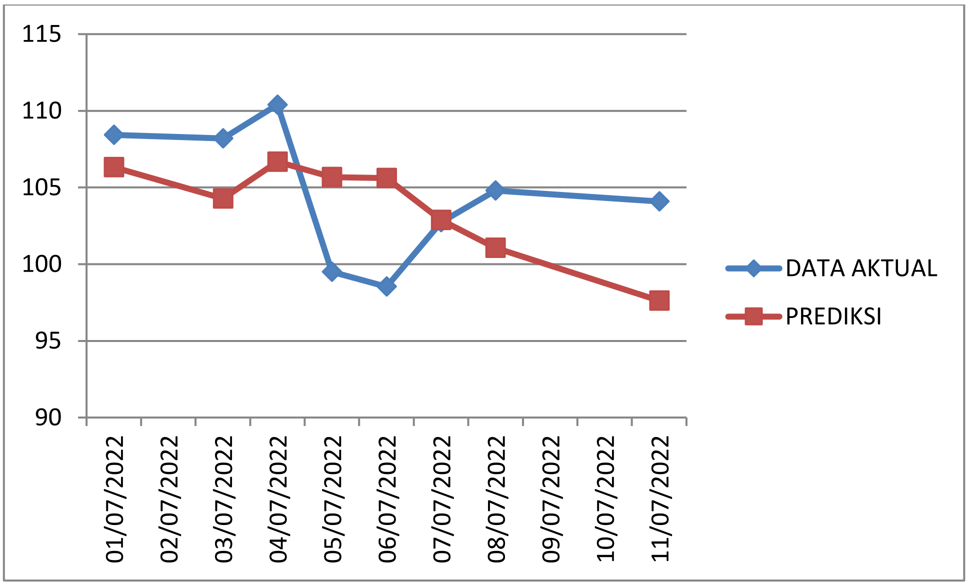


Figure 5
Comparison Chart of Actual and Predicted WTI Crude Oil Prices (1000 iterations)



Model Validation

After obtaining the prediction results for crude oil prices, the MAPE value is then calculated to determine the level of accuracy of the GBM model in predicting crude oil prices. The results of calculating the MAPE value are shown in table 4 below.

Table 4
Accuracy Level of WTI Crude Oil Prediction Results

Iteration	MAPE Value (%)
100	6.092415
200	7.364198
1000	7.276606

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The accuracy level value above shows that the results of predicting crude oil prices using the GBM model are in the good category.

4. Conclusion

The results of crude oil price predictions using the GBM model with 100, 200 and 1000 iterations respectively produce MAPE values of 6.092415%, 7.364198% and 7.276606%. The results show that price prediction by applying the Geometric Brownian Motion model produces a very good level of accuracy. Prediction results with 100 iterations produce the best level of accuracy. Based on the prediction results, it can be seen that the minimum price of crude oil is 96.03$ per barrel and the maximum price of crude oil is 110.43$ per barrel in the period 01 July 2022 to 03 August 2022.

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